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FLUTTER OF A VISCOELASTIC PLATE†

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The problem of the dynamic stability of a rectangular viscoelastic plate (strip) around which there is a flow from one side of a plane-parallel stream of an ideal fluid is considered in a classical formulation. The loading is determined within the framework of piston theory. Attention is paid mainly to investigating the effect of low viscosity on the value of the critical parameter. A traditional stability criterion is used. ©1996 Elsevier Science Ltd. All rights reserved.

In the problem of flutter of a rectangular viscoelastic plate using the Bubnov–Galerkin method and averaging, it was shown in [1, 2] that the critical flow velocity is approximately half that for the corresponding elastic plate with an instantaneous Young's modulus, and this ratio is independent of the viscous properties of the plate material. This result gives rise to a natural dissatisfaction since it refers to the asymptotic stability and it is presented as almost obvious that the sufficient condition of stability and the critical flow velocity corresponding to it can be found from the solution of an elastic problem by replacing the instantaneous modulus by its limiting value. There are no exact solutions of problems on panel flutter which would confirm or refute this intuitive conclusion. It is confirmed in [3] using the example of a single model problem and in this paper using examples of problems on the flutter of a strip and a rectangular plate.

1. FORMULATION OF THE PROBLEM

We consider a rectangular plate which occupies the domain $0 \leqslant x \leqslant l/\beta$, $0 \leqslant y \leqslant l$ in the xy plane The plate is made of a linear viscoelastic material and the relation between the stresses and strain is given by

$$\sigma = E_0(\varepsilon(t) - \lambda \int\limits_{-\infty}^{t} \Gamma(t-\tau)\varepsilon(\tau)d\tau) = E_0(1-\lambda\hat{\Gamma})\varepsilon(t)$$

$$\hat{\Gamma}\varepsilon(t) = \int\limits_{0}^{\infty} \Gamma(\tau)\varepsilon(t-\tau)d\tau$$

Here E_0 is the instantaneous modulus and $\Gamma(t)$ is the relaxation kernel. The limiting value of the modulus is equal to

$$E_\infty = E_0(1-\lambda\Gamma_0), \quad \Gamma_0 = \int\limits_{0}^{\infty} \Gamma(t)dt$$

Henceforth, we shall assume that the material possesses a low viscosity to which $(\lambda\Gamma_0)^2 \ll 1$ corresponds.

The plate is located in a gas stream, the velocity vector of which $\mathbf{V} = \{\upsilon_x, \upsilon_y\}$, $|\mathbf{V}| = \upsilon$ is parallel to its plane. It is required to find that value of the flow velocity υ^* for which the motion of the plate will be asymptotically stable, subject to the condition that $\upsilon < \upsilon^*$.

We now introduce, while retaining the same notation for them, the dimensionless coordinates x/l, y/l, the velocity V/C_0 and the time t/t_0 ($t_0^2 = \rho h l^4/D_0$, $D_0 = E_0 h^3/[12(1-\nu^2)]$, p_0, C_0 are the pressure and the velocity of sound in the unperturbed flow, κ is the polytropic index and ρ and ν are the density and Poisson's ratio of the plate material, and h is its thickness). In accordance with the results obtained previously [4], we shall describe the motion of the plate by the equation

$$(1-\lambda\hat{\Gamma})\Delta^2 w + a_2 \mathbf{V}\,\mathrm{grad}\, w + a_1 \partial w/\partial t + \partial^2 w/\partial t^2 = 0 \tag{1.1}$$

$$a_1 = p_0 \kappa l^4/(C_0 D_0 t_0), \quad a_2 = p_0 \kappa l^3/D_0$$

The corresponding boundary conditions have to supplement this equation.

We shall investigate the motion of the plate in the class of functions $w = \varphi(x, y)\exp(i\omega t)$, and, on substituting this into (1.1), we obtain

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$$(1-\lambda\Gamma(\omega))\Delta^2\varphi + a_2 \mathbf{V}\,\mathrm{grad}\,\varphi + (ia_1\omega - \omega^2)\varphi = 0 \tag{1.2}$$

$$\Gamma(\omega) = \int_0^\infty \Gamma(\tau)e^{-i\omega\tau}d\tau = \int_0^\infty \Gamma(\tau)(\cos\omega\tau - i\sin\omega\tau)d\tau \equiv \Gamma_c(\omega) - i\Gamma_s(\omega) \tag{1.3}$$

Motions with Im $\omega > 0$ will be stable and those with Im $\omega < 0$ will be unstable. Real frequencies correspond to the boundary of these domains, and the critical flow velocity is determined from this very condition. We note that, when Im $\omega = 0$, the functions $\Gamma_c(\omega)$ and $T_s(\omega)$ from (1.3) are the cosine and sine Fourier transforms of the relaxation kernel.

2. AN INFINITELY LONG STRIP

In the case when $\upsilon_y = 0$, we take the function $\varphi(x, y)$ which satisfies the conditions of hinged support along the edges $y = 0, y = 1$ in the form [5, 6]: $\varphi(x, y) = \sin \pi y \exp(-i\alpha x)$, where α is a real number which guarantees the boundedness of the initial deflections for all values of x.

Substituting into (1.2), we arrive at the two equations

$$\omega^2 = (1-\lambda\Gamma_c(\omega))(\alpha^2+\pi^2)^2, \quad \frac{a_2\upsilon_x}{a_1} = \frac{\omega}{\alpha} + \lambda\Gamma_s(\omega)\frac{(\alpha^2+\pi^2)^2}{a_1\alpha} \tag{2.1}$$

It follows that $\omega = \omega(\alpha)$, is determined from the first equation, this is substituted into the second equation, and the critical flow velocity $\upsilon_x^* = \min_\alpha(\upsilon_x(\upsilon_x(\alpha))$ and the wave formation parameter α^* corresponding to it are then determined.

In practice, the solution of the problem is complicated by the fact that the analytic solution of the first of Eqs (2.1) in the case of an arbitrary kernel is impossible.

However, by virtue of the obvious inequalities

$$\Gamma_c(\omega) \leqslant \Gamma_0 = \Gamma_c(0), \quad (\lambda\Gamma_c(\omega))^2 \leqslant (\lambda\Gamma_0)^2 \ll 1$$

its solution can be obtained by the converging method of approximations

$$\omega_0 = \alpha^2 + \pi^2, \quad \omega_n = \omega_0(1-\lambda\Gamma_c(\omega_{n-1}))^{1/2}, \quad n = 1, 2, \ldots$$

whence the representation $\omega_n = \omega_0(1 - \lambda\Gamma_e(\omega_0))^{1/2} + \lambda^2 A_n(\omega_0, \lambda)$, in which $A_n(\omega_0, \lambda)$ are bounded functions, follows, assuming a continuous derivative $\Gamma_c(\omega_0)$ exists. We restrict ourselves to the first term, put $\omega = \omega_n \cong \omega_0(1 - \lambda\Gamma_c(\omega_0))^{1/2}$ and substitute this into the second expression from (2.1), retaining terms containing λ to the first power. We obtain

$$\frac{a_2}{a_1}\upsilon_x = \frac{\alpha^2+\pi^2}{\alpha}(1-\lambda\Gamma_c(\omega_0))^{1/2} + \lambda\Gamma_s(\omega_0)\frac{(\alpha^2+\pi^2)^2}{a_1\alpha} \tag{2.2}$$

Assuming a continuous derivative $\Gamma_s'(\omega_0)$ exists, from (2.2) we find the minimum point in the form $\alpha_0 = \pi + \lambda B(\lambda)$, where $B(\lambda)$ is a bounded function. Substituting (2.2), we obtain

$$\frac{a_2}{a_1}\upsilon_x^* = 2\pi(1-\lambda\Gamma_c(2\pi^2))^{1/2} + \lambda\Gamma_s(2\pi^2)\frac{4\pi^3}{a_1} \tag{2.3}$$

apart from terms in the first power of λ.

We now introduce the "limiting modulus" value of the velocity $\upsilon_0 = (2\pi a_1/a_2)(1 - \lambda\Gamma_0)^{1/2}$. It is obvious that $\upsilon_0 < \upsilon_x^*$ and, hence, when $\upsilon_x < \upsilon_0$, $\upsilon_x < \upsilon_x^*$ will certainly hold. Consequently, the inequality $\upsilon_x < \upsilon_0$ will be a sufficient condition for stable motions of a viscoelastic strip.

In the case when $\upsilon_x = 0$, we construct the approximate solution of (1.2) using the Bubnov–Galerkin method in the two-term approximation (this turns out to be sufficient to achieve good accuracy [7] when solving problems of the flutter of elastic plates)

$$\varphi(x, y) = (C_1 \sin\pi y + C_2 \sin 2\pi y)\exp(-i\alpha x)$$

After carrying out a well-known procedure, we obtain the system of equations

$$\omega^3 + \frac{\lambda_1+\lambda_2}{2a_1}\lambda\Gamma_s(\omega)\omega^2 - \frac{\lambda_1+\lambda_2}{2}(1-\lambda\Gamma_c(\omega))\omega - \frac{\lambda_1\lambda_2}{a_1}(1-\lambda\Gamma_c(\omega))\lambda\Gamma_s(\omega) = 0 \tag{2.4}$$

$$\left(\frac{8}{3}a_2\upsilon_y\right)^2 = [a_1^2 + (\lambda_1+\lambda_2)(1-\lambda\Gamma_c(\omega))]\omega^2 + \lambda\Gamma_s(\omega)(\lambda_1+\lambda_2)a_1\omega -$$
$$-\lambda_1\lambda_2[(1-\lambda\Gamma_c(\omega))^2 - \lambda^2\Gamma_s^2(\omega)] - \omega^4 \tag{2.5}$$
$$\lambda_1 = \alpha^2+\pi^2, \quad \lambda_2 = \alpha^2 + 4\pi^2$$

As in the previous case, it is necessary to determine $\omega = \omega(\alpha)$ from the first equation, substitute it into the second and then minimize it with respect to α.

We denote the left-hand side of (2.4) by $f(\omega)$ and let ω^* be the positive root of the equation $2\omega^2 = (\lambda_1 + \lambda_2)(1 - \lambda\Gamma_c(\omega))$. The existence of this root is proved in the same way as before.

It is easily shown that $f(0) = 0, f'(0) < 0, f(\omega^*) > 0$ and all this points to the existence of a positive root of Eq. (2.4) which is smaller than ω^*. We find this root using the expansion with respect to the parameter λ. $\omega = \omega^* - \lambda\omega_1 + \ldots$, retaining the terms which have been written out. The calculations lead to the result $\omega_1 = \Gamma_s(\omega^*)(\lambda_2 - \lambda_1)^2/[4a_1(\lambda_1+\lambda_2)]$. On substituting $\omega \cong \omega^* - \lambda\omega_1$ into Eq. (2.5), we finally obtain (with an accuracy, with respect to λ, up to the first power)

$$\left(\frac{8}{3}a_2\upsilon_y^*\right)^2 = B_1(\alpha) + \lambda B_2(\alpha) \tag{2.6}$$

$$B_1(\alpha) = a_1^2\frac{\lambda_1+\lambda_2}{2}(1-\lambda\Gamma_c(\omega^*)) + \frac{(\lambda_2-\lambda_1)^2}{4}(1-\lambda\Gamma_c(\omega^*))^2$$

$$B_2(\alpha) = \Gamma_s(\omega^*)\frac{a_1(1-\lambda\Gamma_c(\omega^*))^{1/2}}{2\sqrt{2}}\frac{(\lambda_1+\lambda_2)^2+4\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^{1/2}}$$

In this case, it is obvious that $B_2(\alpha) > 0$ when $\alpha \geqslant 0$, $\min B_1(\alpha) = B_1(0)$.

If the "limiting modulus" velocity υ_0 is now introduced

$$\upsilon_0^2 = \frac{17}{2}\left(\frac{3a_1\pi^2}{8a_2}\right)^2(1-\lambda\Gamma_0) + \left[\frac{45\pi^4(1-\lambda\Gamma_0)}{16a_2}\right]^2$$

then it is possible to assert that the inequality $\upsilon_y < \upsilon_0$ will be a sufficient condition for stable motions of the strip.

3. A RECTANGULAR PLATE

Assuming that $\upsilon_x = 0$, we take the two-term approximation

$$\varphi(x,y) = (C_1\sin\pi y + C_2\sin 2\pi y)\sin\beta\pi x$$

for $\varphi(x, y)$.

The Bubnov–Galerkin procedure with respect to (1.3) leads to system (2.4), (2.5) in which the parameters λ_1 and λ_2 have to be replaced by $\beta_1 = \pi^4(1+\beta^2)^2$ and $\beta_2 = \pi^4(4+\beta^2)^2$, respectively.

By repeating, apart from the new notation, the arguments and calculations of Section 2, we obtain a relation for the critical velocity which is analogous to (2.6), where $2\omega^{*2} = (\beta_1+\beta_2)(1-\lambda\Gamma_c(\omega^*))$. We now introduce the "limiting modulus" velocity υ_0.

$$\upsilon_0^2 = \left(\frac{3a_1}{8\sqrt{2}a_2}\right)^2(\beta_1+\beta_2)(1-\lambda\Gamma_0) + \left[\frac{3(\beta_2-\beta_1)(1-\lambda\Gamma_0)}{16a_2}\right]^2$$

It is obvious that the inequality $\upsilon_y > \upsilon_0$ will be the sufficient condition for stable vibrations of the plate.

Hence, the critical velocity of the flutter of a viscoelastic plate (with "low" viscosity) depends continuously on the viscosity parameter and can be found (conservatively) as the limiting modular velocity by solving the corresponding elastic problem. The possibility of using the Bubnov–Galerkin and averaging methods jointly in problems of the non-conservative stability of viscoelastic plates requires a deeper analysis.

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